Dublin City Schools Mathematics Graded Course of Study
A.P. Calculus AB

Course Description: AP Calculus AB is an extension of advanced mathematical concepts studied in Precalculus. Topics include continuity and limits, composite functions, and graphing. An overview of analysis topics of derivatives and integration is presented with emphasis on application. This course is based upon most current Advanced Placement program syllabus for Calculus AB and prepares students for Advanced Placement examination. Students will be encouraged to take the Advanced Placement test for college credit. A graphing calculator is required.

I. AP Calculus AB Major Topic: **Functions, Graphs, and Limits**

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Topics	AP Calculus AB Indicators
A. Limits of functions (including one-sided limits)	1. An intuitive understanding of the limiting process 2. Calculating limits using algebra 3. Estimating limits from graphs or tables of data
B. Asymptotic and unbounded behavior	1. Understanding asymptotes in terms of graphical behavior 2. Describing asymptotic behavior in terms of limits involving infinity 3. Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)
C. Continuity as a property of functions	1. An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.) 2. Understanding continuity in terms of limits 3. Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

II. AP Calculus AB Major Topic: **Derivatives**

Topics	AP Calculus AB Indicators
A. Concept of the derivative	1. Derivative presented graphically, numerically, and analytically 2. Derivative interpreted as an instantaneous rate of change 3. Derivative defined as the limit of the difference quotient 4. Relationship between differentiability and continuity
B. Derivative at a point	1. Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents. 2. Tangent line to a curve at a point and local linear approximation. 3. Instantaneous rate of change as the limit of average rate of change 4. Approximate rate of change from graphs and tables of values
C. Derivative as a function	1. Corresponding characteristics of graphs of f and f'. 2. Relationship between the increasing and decreasing behavior of f and the sign of f'. 3. The Mean Value Theorem and its geometric interpretation 4. Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
D. Second derivatives	1. Corresponding characteristics of the graphs of f, f', and f'' 2. Relationship between the concavity of f and the sign of f' 3. Points of inflection as places where concavity changes
E. Applications of derivatives	1. Analysis of curves, including the notions of monotonicity and concavity 2. Optimization, both absolute (global) and relative (local) extrema

	3. Modeling rates of change, including related rates problems 4. Use of implicit differentiation to find the derivative of an inverse function 5. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration 6. Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
F. Computation of derivatives	1. Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions 2. Derivative rules for sums, products, and quotients of functions 3. Chain rule and implicit differentiation

III. AP Calculus AB Major Topic: **Integrals**

Topics	AP Calculus AB Indicators
A. Interpretations and properties of definite integrals	1. Definite integral as a limit of Riemann sums 2. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx = f(b) - f(a)$ 3. Basic properties of definite integrals (examples include additivity and linearity)
B. Applications of integrals	1. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. 2. Write an approximating Riemann sum and represent its limit as a definite integral. 3. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.
C. Fundamental Theorem of Calculus	1. Use of the Fundamental Theorem to evaluate definite integrals 2. Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
D. Techniques of antidifferentiation	1. Antiderivatives following directly from derivatives of basic functions 2. Antiderivatives by substitution of variables (including change of limits for definite integrals).

E. Applications of antidifferentiation	1. Finding specific antiderivatives using initial conditions, including applications to motion along a line 2. Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth) 3. Interpretation of slope fields and their application to differential equations.
F. Numerical approximations to definite integrals	1. Use of Riemann sums (using left, right, and midpoint evaluation points) to approximate definite integrals of functions represented algebraically, graphically, and by tables of values 2. Use of trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

IV. Content Standard: **Mathematical Processes**

The benchmarks for mathematical processes articulate what students should demonstrate in problem solving, representation, communication, reasoning and connections at key points in their mathematics program. Specific grade-level indicators have not been included for the mathematical processes standard because content and processes should be interconnected at the indicator level. Therefore, mathematical processes have been embedded within the grade-level indicators for the five content standards

A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.

B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.

C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the *x*-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.

D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.

E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

G. Write clearly and coherently about mathematical thinking and ideas.

H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Dublin City Schools Mathematics Graded Course of Study
A.P. Calculus BC

Course Description: Topics for AP Calculus BC include functions, graphs, and limits; derivatives, integrals and polynomial approximations and series using a numerical, graphical, analytical, and verbal approach. Derivatives will include the analysis of planar curves given in parametric, polar, and vector form including velocity and acceleration vectors. There will be a geometric interpretation of differential equations via slope fields and the relationship between slope fields and derivatives of implicitly defined functions. The course will also include numerical solution of differential equations using Euler's Method. The concept of polynomial approximations and series will include the series of constants and Taylor Series. This course is based upon most current Advanced Placement program syllabus for Calculus BC and prepares students for Advanced Placement examination. Students are encouraged to take the AP exam. A graphing calculator is required.

I. AP Calculus BC Major Topic: **Functions, Graphs, and Limits**

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Topics	AP Calculus BC Indicators
A. Limits of functions (including one-sided limits)	1. An intuitive understanding of the limiting process 2. Calculating limits using algebra 3. Estimating limits from graphs or tables of data
B. Asymptotic and unbounded behavior	1. Understanding asymptotes in terms of graphical behavior 2. Describing asymptotic behavior in terms of limits involving infinity 3. Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)
C. Continuity as a property of functions	1. An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.) 2. Understanding continuity in terms of limits 3. Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

D. Parametric, polar, and vector functions	1. The analysis of planar curves in parametric form, 2. The analysis of planar curves in polar form 3. The analysis of planar curves in vector form.

II. AP Calculus BC Major Topic: **Derivatives**

Topics	AP Calculus BC Indicators
A. Concept of the derivative	1. Derivative presented graphically, numerically, and analytically 2. Derivative interpreted as an instantaneous rate of change 3. Derivative defined as the limit of the difference quotient 4. Relationship between differentiability and continuity
B. Derivative at a point	1. Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents. 2. Tangent line to a curve at a point and local linear approximation. 3. Instantaneous rate of change as the limit of average rate of change 4. Approximate rate of change from graphs and tables of values
C. Derivative as a function	1. Corresponding characteristics of graphs of f and f'. 2. Relationship between the increasing and decreasing behavior of f and the sign of f'. 3. The Mean Value Theorem and its geometric interpretation 4. Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
D. Second derivatives	1. Corresponding characteristics of the graphs of f, f', and f'' 2. Relationship between the concavity of f and the sign of f' 3. Points of inflection as places where concavity changes

E. Applications of derivatives	1. Analysis of curves, including the notions of monotonicity and concavity 2. Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration 3. Optimization, both absolute (global) and relative (local) extrema 4. Modeling rates of change, including related rates problems 5. Use of implicit differentiation to find the derivative of an inverse function 6. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration 7. Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations 8. Numerical solution of differential equations using Euler's method 9. L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series
F. Computation of derivatives	1. Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions 2. Derivative rules for sums, products, and quotients of functions 3. Chain rule and implicit differentiation 4. Derivatives of parametric, polar, and vector functions

Dublin City Schools Mathematics Graded Course of Study
A.P. Calculus BC

III. AP Calculus BC Major Topic: **Integrals**

Topics	AP Calculus BC Indicators
A. Interpretations and properties of definite integrals	1. Definite integral as a limit of Riemann sums 2. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx = f(b) - f(a)$ 3. Basic properties of definite integrals (examples include additivity and linearity)
B. Applications of integrals	1. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. 2. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. 3. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral.

	4. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.
C. Fundamental Theorem of Calculus	1. Use of the Fundamental Theorem to evaluate definite integrals 2. Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
D. Techniques of antidifferentiation	1. Antiderivatives following directly from derivatives of basic functions 2. Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only) 3. Improper integrals (as limits of definite integrals)
E. Applications of antidifferentiation	1. Finding specific antiderivatives using initial conditions, including applications to motion along a line 2. Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth) 3. Solving logistic differential equations and using them in modeling

F. Numerical approximations to definite integrals	1. Use of Riemann sums (using left, right, and midpoint evaluation points) to approximate definite integrals of functions represented algebraically, graphically, and by tables of values 2. Use of trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

IV. AP Calculus BC Major Topic: **Polynomial Approximations and Series**

Topics	AP Calculus BC Indicators
A. Concept of series	1. A series is defined as a sequence of partial sums, and 2. Convergence is defined in terms of the limit of the sequence of partial sums. 3. Technology can be used to explore convergence and divergence.
B. Series of constants	1. Motivating examples, including decimal expansion 2. Geometric series with applications 3. The harmonic series 4. Alternating series with error bound 5. Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series 6. The ratio test for convergence and divergence 7. Comparing series to test for convergence or divergence
C. Taylor series	1. Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve) 2. Maclaurin series and the general Taylor series centered at x = a 3. Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\frac{1}{1-x}$.

	4. Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series 5. Functions defined by power series 6. Radius and interval of convergence of power series 7. Lagrange error bound for Taylor polynomials

V. Content Standard: **Mathematical Processes**

The benchmarks for mathematical processes articulate what students should demonstrate in problem solving, representation, communication, reasoning and connections at key points in their mathematics program. Specific grade-level indicators have not been included for the mathematical processes standard because content and processes should be interconnected at the indicator level. Therefore, mathematical processes have been embedded within the grade-level indicators for the five content standards

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C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.

D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.

E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

G. Write clearly and coherently about mathematical thinking and ideas.

H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.