K-12 Mathematics Education Vision

In Dublin City Schools, we believe that *all students* deserve a mathematical learning experience centered around communication, collaboration, thinking and problem solving.

We believe that our students will become mathematicians through opportunities to:

- approach mathematics with curiosity, courage, confidence & intuition.
- think flexibly, critically and creatively with numbers and problems.
- take risks and persevere through robust problem solving.
- use math as a means to show the interconnectedness of our world.
- develop a mathematical mindset that emphasizes the importance of understanding and communicating process, while also providing precise answers.
- engage in mathematical discourse as the language of problem solving and innovative thinking.

This experience will prepare our students for college, career, and life as innovative thinkers and problem solvers of the future.

Instructional Agreements for Mathematical Learning within the Dublin City Schools

1. Learning goals will be communicated to guide students through the expectations of mathematical learning using a variety of instructional techniques and technology integration.
2. Teachers will ensure a safe, challenging learning environment in which students experience a balance of independent and collaborative learning, while supporting productive stretch for all students.
3. Instruction will support students in using and connecting mathematical representations.
4. Procedural fluency will be built from student conceptual understanding.
5. Content standards will be learned in partnership with the 8 Mathematical Practices.

Dublin City Schools Mathematics
Graded Course of Study

K-12 Mathematical Practices:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x^2 + x + 1), and (x – 1)(x^3 + x^2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Dublin City Schools Mathematics
Graded Course of Study

PRECALCULUS and HONORS PRECALCULUS

Precalculus and Honors Precalculus Course Goals:

Precalculus integrates algebra, geometry, and trigonometry and is aligned to the plus (fourth course) standards of Ohio's Math Learning Standards. Mathematicians in this course will build upon prior learning and continue to explore radical, exponential, logarithmic, and trigonometric equations and functions. Students will also investigate vectors, conic sections, sequences and series, and applications of matrices as they prepare for studies in Calculus.

Course Content Standards:

Domain	Cluster	Standard
VECTOR AND MATRIX QUANTITIES	Represent and model with vector quantities	**N.VM.1(+)** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||)
		N.VM.2(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
		N.VM.3(+) Solve problems involving velocity and other quantities that can be represented by vectors
	Perform operations on vectors	**N.VM.4(+)** Add and subtract vectors **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **c.** Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
		N.VM.5(+) Multiply a vector by a scalar. **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx , vy) = (cvx , cvy).

		b. Compute the magnitude of a scalar multiple cv using cv = |c|v. Compute the direction of cv knowing that when cv ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c <0)
	Perform operations on matrices, and use matrices in applications.	**N.VM.6(+)** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
		N.VM.7(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
		N.VM.8(+) Add, subtract, and multiply matrices of appropriate dimensions.
		N.VM.9(+) Understand that, unlike multiplication of numbers, matrix multiplication of square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
		N.VM.10(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication analogous to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
		N.VM.11(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
		N.VM.12(+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
SEEING STRUCTURE IN EXPRESSIONS	Interpret the structure of expressions	**A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$. For example, $\log(x+2)^2$ can be rewritten as $2\log(x+2)$.
		DubPC.11 Factor expressions and solve equations that contain rational exponents, e.g. $(x+1)^{3/2} -2(x+1)^{-½}$
		DubPC.14 Utilize logarithmic properties, including change of base, to expand and condense logarithmic expressions.
	Write expressions in equivalent forms to solve problems	**A.SSE.4(+)** Derive a formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
		DubPC.15 Apply properties of exponents and logarithms in order to solve exponential and logarithmic equations.
		DubPC.16 Solve quadratic-like exponential functions by factoring. For example,

		solve $e^{2x} + 5e^{x} + 6 = 0$
		DubPC.20 Define sigma notation and use it to determine partial sums of all kinds of series (ie ones that do not have an obvious explicit formula for the series).
		DubPC.21 Understand the difference between the explicit formula for the nth term and the sigma of that formula.
		DubPC.22 Find the sum of an infinite geometric series. Discuss convergence and divergence.
		DubPC.23 Explore convergent and divergent series other than Arithmetic and Geometric. For example: as n goes to infinity, the Harmonic Series as a decreasing infinite series that is NOT convergent and 1n! an additional convergent infinite series that is NOT geometric.
ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS	Use polynomial identities to solve problems.	**A.APR.5(+)** Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.
REASONING WITH EQUATIONS AND INEQUALITIES	Solving systems of equations	**A.REI.6** Solve systems of linear equations algebraically. **b**. Extend to include solving systems of linear equations in three variables algebraically.
		A.REI.8(+) Represent a system of linear equations as a single matrix equation in a vector variable.
		A.REI.9(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).
	Represent and solve equations and inequalities graphically	**A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.
		DubPC.8 Solve trigonometric equations using a variety of strategies (inverse functions, graphically, using the Unit Circle, algebraically).
		Dub.PC.10 Solve advanced radical equations including all indices. (ie: solving radical equations that involve squaring both sides of the equation twice)
INTERPRETING FUNCTIONS	Interpret functions that arise in applications in terms of the context.	**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and

		periodicity.*
	Analyze functions using different representations.	**F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. **f.** Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Include all 6 parent trigonometric functions. **g.** (+)Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior **h.** (+)Graph logarithmic functions, indicating intercepts and end behavior.
		DubPC.2 Recognize even and odd functions from their graphs and algebraic expressions for them.
		DubPC.9 Graph radical functions beyond transformations on the parent functions for the square root and cube root functions.
		DubPC.12 Find the equations of slant asymptotes and coordinates of holes for rational functions.
BUILDING FUNCTIONS	Build a function that models a relationship between two quantities	**F.BF.1** Write a function that describes a relationship between two quantities.* **c.** **(+)** Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
	Build new functions from existing functions	**F.BF.3** Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x+k) for specific values of K (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
		F.BF.4(+) Find inverse functions. **c.** Verify by composition that one function is the inverse of another. **d.** Find the inverse of a function algebraically, given that the function has an inverse. **e.** Produce an invertible function from a non-invertible function by restricting the domain.
		F.BF.5(+) Understand the inverse relationship between exponents and logarithms

		and use this relationship to solve problems involving logarithms and exponents.
		DubPC.1 Combine functions using addition, subtraction, multiplication, and division both graphically and analytically.
		DubPC.3 Given h(x) find two functions f and g such that f(g(x)) = h(x).
LINEAR, QUADRATIC AND EXPONENTIAL MODELS	Construct and compare linear, quadratic, and exponential models, and solve problems.	**DubPC.17** Model with exponential and logarithmic functions. Including but not limited to compound interest models.
TRIGONOMETRIC FUNCTIONS	Extend the domain of trigonometric functions using the unit circle.	**F.TF.3(+)** Use special right triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi/4 and pi/6, and use the unit circle to express the values of sine, cosine, and tangent for pi-x, pi+x, and 2pi-x in terms of their values for x, where x is any real number (AKA reference angles).
		F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
		DubPC.4 Evaluate the six trigonometric ratios and their inverses using tools such as the graphs of the functions and inverse functions, or the Unit Circle.
	Model periodic phenomena with trigonometric functions.	**F.TF.6 (+)** Understand that restricting a trigonometric function to a domain on which it is always increasing or decreasing allows its inverse to be constructed.
		F.TF.7(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*
		DubPC.5 Given a graph of a trigonometric function, recognize that many equations exist and write an appropriate model.
	Prove and apply trigonometric identities.	**F.TF.8** Prove the Pythagorean identity and use it to find sin(x), cos(x), or tan (x) and the quadrant of the angle.
		F.TF.9(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
		DubPC.6 Verify trigonometric identities utilizing the fundamental trigonometric identities, pythagorean, sum/difference and double angle identities and work flexibly within trigonometric expressions and equations.
		DubPC.7 Find exact values of combinations of angles using sum and difference formulas and double angle formulas.
SIMILARITY, RIGHT TRIANGLES, AND	Apply trigonometry to general triangles.	**G.SRT.9(+)** Derive the formula A=(1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

TRIGONOMETRY		**G.SRT.10(+)** Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.
		G.SRT.11(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS	Translate between the geometric description and the equation for a conic section.	**G.GPE.2(+)** Derive the equation of a parabola given a focus and directrix.
		G.GPE.3(+) Derive the equation of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
		DubPC.24 Sketch a graph of a circle, parabola, ellipse, and hyperbola using paper and pencil and technology.