



## **K-12 Mathematics Education Vision**

In Dublin City Schools, we believe that *all students* deserve a mathematical learning experience centered around communication, collaboration, thinking and problem solving.

We believe that our students will become mathematicians through opportunities to:

- approach mathematics with curiosity, courage, confidence & intuition.
- think flexibly, critically and creatively with numbers and problems.
- take risks and persevere through robust problem solving.
- use math as a means to show the interconnectedness of our world.
- develop a mathematical mindset that emphasizes the importance of understanding and communicating process, while also providing precise answers.
- engage in mathematical discourse as the language of problem solving and innovative thinking.

This experience will prepare our students for college, career, and life as innovative thinkers and problem solvers of the future.

## **Instructional Agreements for Mathematical Learning within the Dublin City Schools**

1. Learning goals will be communicated to guide students through the expectations of mathematical learning using a variety of instructional techniques and technology integration.
2. Teachers will ensure a safe, challenging learning environment in which students experience a balance of independent and collaborative learning, while supporting productive stretch for all students.
3. Instruction will support students in using and connecting mathematical representations.
4. Procedural fluency will be built from student conceptual understanding.
5. Content standards will be learned in partnership with the 8 Mathematical Practices.

# Dublin City Schools Mathematics Graded Course of Study

## **K-12 Mathematical Practices:**

### **1. Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### **2. Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### **3. Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

# Dublin City Schools Mathematics Graded Course of Study

## 4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## 7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see

## Dublin City Schools Mathematics Graded Course of Study

complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

### **8. Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

# Dublin City Schools Mathematics Graded Course of Study

## MATH 7

### Math 7 Course Goals:

Mathematicians in this course focus on the following critical areas, while incorporating the eight mathematical practices: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; (4) drawing inferences about populations based on samples; and (5) investigating chance processes and develop, use and evaluate probability models. Learners will apply their mathematical understanding in real world context, making meaning of math in their worlds.

### Course Content Standards:

Domain	Cluster	Standard
RATIOS AND PROPORTIONAL RELATIONSHIPS	Analyze proportional relationships and use them to solve real-world and mathematical problems.	<b>7.RP.1</b> Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. <i>For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.</i>
		<b>7.RP.2</b> Recognize and represent proportional relationships between quantities. <b>a.</b> Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. <b>b.</b> Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. <b>c.</b> Represent proportional relationships by equations. <i>For example, if total cost <math>t</math> is proportional to the number <math>n</math> of items purchased at a constant price <math>p</math>, the relationship between the total cost and the number of items can be expressed as <math>t = pn</math>.</i> <b>d.</b> Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.
		<b>7.RP.3</b> Use proportional relationships to solve multistep ratio and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i>
THE NUMBER SYSTEM	Apply and extend previous understandings of operations with fractions to add, subtract, multiply,	<b>7.NS.1</b> Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

## Dublin City Schools Mathematics Graded Course of Study

	and divide rational numbers.	<p><b>a.</b> Describe situations in which opposite quantities combine to make 0. <i>For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.</i></p> <p><b>b.</b> Understand <math>p + q</math> as the number located a distance <math> q </math> from <math>p</math>, in the positive or negative direction depending on whether <math>q</math> is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.</p> <p><b>c.</b> Understand subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</p> <p><b>d.</b> Apply properties of operations as strategies to add and subtract rational numbers</p> <p><b>7.NS.2</b> Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.</p> <p><b>a.</b> Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as <math>(-1)(-1) = 1</math> and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.</p> <p><b>b.</b> Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If <math>p</math> and <math>q</math> are integers, then <math>-(p/q) = (-p)/q = p/(-q)</math>. Interpret quotients of rational numbers by describing real-world contexts.</p> <p><b>c.</b> Apply properties of operations as strategies to multiply and divide rational numbers.</p> <p><b>d.</b> Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p> <p><b>7.NS.3</b> Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.</p>
EXPRESSIONS AND EQUATIONS	Use properties of operations to generate equivalent expressions.	<p><b>7.EE.1</b> Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</p> <p><b>7.EE.2</b> In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. <i>For example,</i></p>

## Dublin City Schools Mathematics Graded Course of Study

	<p>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p>	<p><i>a discount of 15% (represented by <math>p - 0.15p</math>) is equivalent to <math>(1 - 0.15)p</math>, which is equivalent to <math>0.85p</math> or finding 85% of the original price.</i></p> <p><b>7.EE.3</b> Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example, if a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar <math>9\frac{3}{4}</math> inches long in the center of a door that is <math>27\frac{1}{2}</math> inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i></p> <p><b>7.EE.4</b> Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p><b>a.</b> Solve word problems leading to equations of the form <math>px + q = r</math> and <math>p(x + q) = r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. <i>For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</i></p> <p><b>b.</b> Solve word problems leading to inequalities of the form <math>px + q &gt; r</math> or <math>px + q &lt; r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. <i>For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</i></p>
<p>GEOMETRY</p>	<p>Draw, construct, and describe geometrical figures and describe the relationships between them.</p>	<p><b>7.G.1</b> Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.</p> <p><b>a.</b> Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.</p> <p><b>b.</b> Represent proportional relationships within and between similar figures.</p> <p><b>7.G.2</b> Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions.</p> <p><b>a.</b> Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p>

## Dublin City Schools Mathematics Graded Course of Study

		<p><b>b.</b> Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions</p> <p><b>7.G.3</b> Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</p>
	Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.	<p><b>7.G.4</b> Work with circles.</p> <p><b>a.</b> Explore and understand the relationships among the circumference, diameter, area, and radius of a circle.</p> <p><b>b.</b> Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.</p> <p><b>7.G.5</b> Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</p> <p><b>7.G.6</b> Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p>
STATISTICS AND PROBABILITY	Use sampling to draw conclusions about a population.	<p><b>7.SP.1</b> Understand that statistics can be used to gain information about a population by examining a sample of the population.</p> <p><b>a.</b> Differentiate between a sample and a population.</p> <p><b>b.</b> Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.</p>
	Broaden understanding of statistical problem solving.	<p><b>7.SP.2</b> Broaden statistical reasoning by using the GAISE model:</p> <p><b>a.</b> Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. <i>For example, "How do the heights of seventh graders compare to the heights of eighth graders?"</i> (GAISE Model, step 1)</p> <p><b>b.</b> Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)</p> <p><b>c.</b> Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)</p> <p><b>d.</b> Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)</p>



## Dublin City Schools Mathematics Graded Course of Study

	<p>Summarize and describe distributions representing one population and draw informal comparisons between two populations.</p>	<p><b>7.SP.3</b> Describe and analyze distributions.</p> <p><b>a.</b> Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point.</p> <p><b>b.</b> Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. <i>For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot (line plot), the separation between the two distributions of heights is noticeable.</i></p>
	<p>Investigate chance processes and develop, use, and evaluate probability models.</p>	<p><b>7.SP.5</b> Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around 1/2 indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.</p> <p><b>7.SP.6</b> Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <i>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</i></p> <p><b>7.SP.7</b> Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <p><b>a.</b> Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. <i>For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</i></p> <p><b>b.</b> Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. <i>For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</i></p> <p><b>7.SP.8</b> Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations.</p> <p><b>a.</b> Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound</p>

## Dublin City Schools Mathematics Graded Course of Study

		<p>event occurs.</p> <p><b>b.</b> Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., “rolling double sixes,” identify the outcomes in the sample space which compose the event.</p> <p><b>c.</b> Design and use a simulation to generate frequencies for compound events. <i>For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?</i></p>
--	--	--