



K-12 Mathematics Education Vision

In Dublin City Schools, we believe that *all students* deserve a mathematical learning experience centered around communication, collaboration, thinking and problem solving.

We believe that our students will become mathematicians through opportunities to:

- approach mathematics with curiosity, courage, confidence & intuition.
- think flexibly, critically and creatively with numbers and problems.
- take risks and persevere through robust problem solving.
- use math as a means to show the interconnectedness of our world.
- develop a mathematical mindset that emphasizes the importance of understanding and communicating process, while also providing precise answers.
- engage in mathematical discourse as the language of problem solving and innovative thinking.

This experience will prepare our students for college, career, and life as innovative thinkers and problem solvers of the future.

Instructional Agreements for Mathematical Learning within the Dublin City Schools

1. Learning goals will be communicated to guide students through the expectations of mathematical learning using a variety of instructional techniques and technology integration.
2. Teachers will ensure a safe, challenging learning environment in which students experience a balance of independent and collaborative learning, while supporting productive stretch for all students.
3. Instruction will support students in using and connecting mathematical representations.
4. Procedural fluency will be built from student conceptual understanding.
5. Content standards will be learned in partnership with the 8 Mathematical Practices.

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K-12 Mathematical Practices:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see

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complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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GRADE 4

Grade 4 Mathematics Course Goals:

Fourth grade mathematicians focus on place value, operations of whole numbers and fractions, equivalence of fractions and understanding geometric figures. Mathematicians build an understanding of place value and the relative sizes of numbers in each place, along with fluently adding and subtracting whole numbers up to 1,000,000 using an algorithm. Using place value and properties of operations, students develop fluency with efficient strategies for multiplication and division of multi-digit whole numbers. Fourth graders are able to understand, explain and construct viable arguments to support multiplication and division strategies. Students in this grade will select and accurately apply appropriate methods to estimate and mentally calculate sums, differences, products, and quotients.

Fourth grade mathematicians build on their foundation of composing and decomposing fractions into unit fractions. Students use strategies to generate and recognize equivalent fractions. Students begin to develop methods for adding, subtracting, and multiplying fractions. Fourth graders describe, analyze, compare and classify geometric figures based on their properties to solve problems. Learners will apply their mathematical understanding in real world context, making meaning of math in their worlds.

Course Content Standards:

Domain	Cluster	Standard
OPERATIONS AND ALGEBRAIC THINKING	Use the four operations with whole numbers to solve problems.	4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
		4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
		4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including

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		problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
	Gain familiarity with factors and multiples.	4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.
	Generate and analyze patterns.	4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i>
NUMBER AND OPERATIONS IN BASE TEN	Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.	4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division.
		4.NBT.2 Read and write multi-digit whole numbers using standard form, word form, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
		4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.
	Use place value understanding and properties of operations to perform multi-digit arithmetic with	4.NBT.4 Fluently ^G add and subtract multi-digit whole numbers using a standard algorithm ^G .

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	<p>whole numbers less than or equal to 1,000,000.</p>	<p>4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>
		<p>4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>
<p>NUMBER AND OPERATIONS - FRACTIONS</p>	<p>Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p>	<p>4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>
		<p>4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p>
	<p>Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified).</p>	<p>4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.</p> <p>a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p> <p>b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model^o. <i>Examples:</i> $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.</p> <p>c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition</p>

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		<p>and subtraction.</p> <p>d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</p>
		<p>4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p> <p>a. Understand a fraction a/b as a multiple of $1/b$. <i>For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$ or $5/4 = (1/4) + (1/4) + (1/4) + (1/4) + (1/4)$.</i></p> <p>b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. <i>For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)</i></p> <p>c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i></p>
	<p>Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p>	<p>4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. <i>For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.</i> In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade.</p>

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		<p>4.NF.6 Use decimal notation for fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</i></p>
		<p>4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.</p>
<p>MEASUREMENT AND DATA</p>	<p>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p>	<p>4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. <i>For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...</i></p>
		<p>4.MD.2 Solve real-world problems involving money, time, and metric measurement.</p> <p>a. Using models, add and subtract money and express the answer in decimal notation.</p> <p>b. Using number line diagrams^G, clocks, or other models, add and subtract intervals of time in hours and minutes.</p> <p>c. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.</p>
		<p>4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. <i>For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.</i></p>

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	Represent and interpret Data	4.MD.4 Display and interpret data in graphs (picture graphs, bar graphs, and line plots ^G) to solve problems using numbers and operations for this grade.
	Geometric measurement: understand concepts of angle and measure angles.	4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. <ul style="list-style-type: none"> a. Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles. b. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.
		4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
		4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
GEOMETRY	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	4.G.1 Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
		4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.