K-12 Mathematics Education Vision

In Dublin City Schools, we believe that *all students* deserve a mathematical learning experience centered around communication, collaboration, thinking and problem solving.

We believe that our students will become mathematicians through opportunities to:

- approach mathematics with curiosity, courage, confidence & intuition.
- think flexibly, critically and creatively with numbers and problems.
- take risks and persevere through robust problem solving.
- use math as a means to show the interconnectedness of our world.
- develop a mathematical mindset that emphasizes the importance of understanding and communicating process, while also providing precise answers.
- engage in mathematical discourse as the language of problem solving and innovative thinking.

This experience will prepare our students for college, career, and life as innovative thinkers and problem solvers of the future.

Instructional Agreements for Mathematical Learning within the Dublin City Schools

1. Learning goals will be communicated to guide students through the expectations of mathematical learning using a variety of instructional techniques and technology integration.
2. Teachers will ensure a safe, challenging learning environment in which students experience a balance of independent and collaborative learning, while supporting productive stretch for all students.
3. Instruction will support students in using and connecting mathematical representations.
4. Procedural fluency will be built from student conceptual understanding.
5. Content standards will be learned in partnership with the 8 Mathematical Practices.

K-12 Mathematical Practices:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see

complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x^2 + x + 1), and (x – 1)(x^3 + x^2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Dublin City Schools Mathematics
Graded Course of Study

GEOMETRY

Geometry Course Goals:

Mathematicians in this course will explore complex geometric situations and deepen their explanations of geometric relationships moving towards formal mathematical arguments. This course builds on congruence and similarity concepts introduced in previous courses. Students develop their understanding and use of proof, both formal and informal. Students focus learning in trigonometry, circles, and connecting coordinates to both algebra and geometry concepts. Students further develop concepts in probability, expanding their ability to compute and interpret theoretical and experimental probabilities.

Course Content Standards:

Domain	Cluster	Standard
CIRCLES	Understand and apply theorems about circles.	**G.C.1** Prove that all circles are similar using transformational arguments.
		G.C.2 Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
		G.C.3 Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.
		G.C.4(+) Construct a tangent line from a point outside a given circle to the circle.
	Find arc lengths and areas of sectors of circles.	**G.C.5** Find arc lengths and areas of sectors of circles. **a.** Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. **b.** Derive the formula for the area of a sector, and use it to solve problems.
CONGRUENCE	Experiment with transformations in the plane.	**G.CO.1** Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.
		G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus

		horizontal stretch.
		G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. **a.** Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. **b.** Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.
		G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
		G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another
	Understand congruence in terms of rigid motions	**G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
		G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
		G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
	Prove geometric theorems both formally and informally using a variety of methods.	**G.CO.9** Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
		G.CO.10 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side*
		G.CO.11 Prove and apply theorems about parallelograms. *Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*

	Make geometric constructions.	**G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
		G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
	Classify and analyze geometric figures.	**G.CO.14** Classify two-dimensional figures in a hierarchy based on properties.
GEOMETRIC MEASUREMENT AND DIMENSION	Explain volume formulas, and use them to solve problems	**G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
		G.GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
		G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
	Visualize relationships between two-dimensional and three-dimensional objects.	**G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
	Understand the relationships between lengths, area, and volumes.	**G.GMD.5** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.
		G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2 , and k^3 , respectively.
EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS	Translate between the geometric description and the equation for a conic section.	**G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
	Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.	**G.GPE.4** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.*
		G.GPE.5 Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or

		perpendicular to a given line that passes through a given point.
		G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
		G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
MODELING WITH GEOMETRY	Apply geometric concepts in modeling situations.	**G.MG.1** Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.
		G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.
		G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY	Understand similarity in terms of similarity transformations	**G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor. **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
		G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
		G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
	Prove and apply theorems both formally and informally involving similarity using a variety of methods.	**G.SRT.4** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
		G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.
	Define trigonometric ratios, and solve problems involving right triangles.	**G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

		G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
		G.SRT.8 Solve problems involving right triangles. **a.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. **b.** (+) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY	Understand independence and conditional probability, and use them to interpret data.	**S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
		S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
		S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
		S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
		S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
	Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
		S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.
		(+)S.CP.8 Apply the general Multiplication Rule in a uniform probability model,

		P(A and B) = P(A)•P(B|A) = P(B)•P(A|B), and interpret the answer in terms of the model.
		(+)S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.