K-12 Mathematics Education Vision

In Dublin City Schools, we believe that *all students* deserve a mathematical learning experience centered around communication, collaboration, thinking and problem solving.

We believe that our students will become mathematicians through opportunities to:

- approach mathematics with curiosity, courage, confidence & intuition.
- think flexibly, critically and creatively with numbers and problems.
- take risks and persevere through robust problem solving.
- use math as a means to show the interconnectedness of our world.
- develop a mathematical mindset that emphasizes the importance of understanding and communicating process, while also providing precise answers.
- engage in mathematical discourse as the language of problem solving and innovative thinking.

This experience will prepare our students for college, career, and life as innovative thinkers and problem solvers of the future.

Instructional Agreements for Mathematical Learning within the Dublin City Schools

1. Learning goals will be communicated to guide students through the expectations of mathematical learning using a variety of instructional techniques and technology integration.
2. Teachers will ensure a safe, challenging learning environment in which students experience a balance of independent and collaborative learning, while supporting productive stretch for all students.
3. Instruction will support students in using and connecting mathematical representations.
4. Procedural fluency will be built from student conceptual understanding.
5. Content standards will be learned in partnership with the 8 Mathematical Practices.

DUBLIN CITY SCHOOLS
Committed to Excellence and Equity

K-12 Mathematical Practices:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

ALGEBRA 1

Algebra 1 Course Goals:

Mathematicians in this course will build upon prior knowledge of linear functions to extend algebraic problem solving to quadratic and exponential relationships. Students will engage in methods for analyzing, solving, and modeling with these functions. Students will graph and interpret characteristics of functions and solve both algebraically and graphically. Students will reason with equations and inequalities, with a focus on modeling. This course will include a study of descriptive statistics during which students will display numerical data and summarize it using measures of center and variability. The GAISE model will support students as they interpret the results in a real world context.

Course Content Standards:

Domain	Cluster	Standard
NUMBER AND QUANTITY	Reason quantitatively and use units to solve problems.	**N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
		N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.
		N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
ARITHMETIC WITH POLYNOMIAL AND RATIONAL EXPRESSIONS	Perform arithmetic operations on polynomials.	**A.APR.1** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **a.** Focus on polynomial expressions that simplify to forms that are linear or quadratic.
CREATING EQUATIONS	Create equations that describe numbers or relationships.	**A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. **a.** Focus on applying linear and simple exponential expressions. **b.** Focus on applying simple quadratic expressions.
		A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **a.** Focus on applying linear and simple exponential expressions. **b.** Focus on applying simple quadratic expressions.

		A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
		A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **a.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R, or rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r*
REASONING WITH EQUATIONS AND INEQUALITIES	Understand solving equations as a process of reasoning and explain the reasoning.	**A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
	Solve equations and inequalities in one variable.	**A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
		A.REI.4 Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. **c.(+)** Derive the quadratic formula using the method of completing the square.
	Solve systems of equations.	**A.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
		A.REI.6 Solve systems of linear equations algebraically and graphically. **a.** Limit to pairs of linear equations in two variables.
		A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$
	Represent and solve equations and inequalities graphically.	**A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
		A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;

		find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
		A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
SEEING STRUCTURE IN EXPRESSIONS	Interpret the structure in expressions.	**A.SSE.1** Interpret expressions that represent a quantity in terms of its context. **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.
		A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
	Write expressions in equivalent forms to solve problems.	**A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **a.** Factor a quadratic expression to reveal the zeros of the function it defines. **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8t can be written as 2^{3t}.*
BUILDING FUNCTIONS	Build a function that models a relationship between two quantities	**F.BF.1** Write a function that describes a relationship between two quantities. **a.** Determine an explicit expression, a recursive process, or steps for calculation from context. i. Focus on linear and exponential functions. ii. Focus on situations that exhibit quadratic or exponential relationships.
		F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
	Build new functions from existing functions.	**F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **a.** Focus on transformations of graphs of quadratic functions, except for $f(kx)$

		F.BF.4 Find inverse functions. **a.** Informally determine the input of a function when the output is known.
INTERPRETING FUNCTIONS	Understand the concept of a function, and use function notation.	**F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of f corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.
		F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
		F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.
	Interpret functions that arise in applications in terms of the context.	**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **b.** Focus on linear, quadratic, and exponential functions.
		F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **b.** Focus on linear, quadratic, and exponential functions.
	Analyze functions using different representations.	**F.IF.7** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. **a.** Graph linear functions and indicate intercepts. **b.** Graph quadratic functions and indicate intercepts, maxima, and minima. **e.** Graph simple exponential functions, indicating intercepts and end behavior.
		F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

		i. Focus on completing the square to quadratic functions with the leading coefficient of 1. **b.** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$ *and* $y = (0.97)^t$ *and classify them as representing exponential growth or decay.* **i.** Focus on exponential functions evaluated at integer inputs.
		F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **b.** Focus on linear, quadratic, and exponential functions.
LINEAR, QUADRATIC, AND EXPONENTIAL MODELS	Construct and compare linear, quadratic, and exponential models, and solve problems.	**F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
		F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
		F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
	Interpret expressions for functions in terms of the situation they model.	**F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.
INTERPRETING CATEGORICAL AND QUANTITATIVE DATA	Summarize, represent, and interpret data on a single count or measurement variable	**S.ID.1** Represent data with plots on the real number line (dot plotsG, histograms, and box plots) in the context of real-world applications using the GAISE model.
		S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets.
		S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting

		for possible effects of extreme data points (outliers).
	Summarize, represent, and interpret data on two categorical and quantitative variables.	**S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
		S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **c.** Fit a linear function for a scatterplot that suggests a linear association.
	Interpret linear models.	**S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
		S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.